

# UNCLASSIFIED

AD NUMBER
AD863433
NEW LIMITATION CHANGE
TO Approved for public release, distribution unlimited
FROM Distribution authorized to U.S. Gov't. agencies and their contractors; Administrative/Operational Use; AUG 1969. Other requests shall be referred to Foreign Technology Division, Attn: TDBDR, Wright-Patterson AFB, OH 45433.
AUTHORITY
FTD, USAF ltr, 9 Nov 1971

THIS PAGE IS UNCLASSIFIED

AD 863433

FTD-HT-23-1328-68

## FOREIGN TECHNOLOGY DIVISION



### A MODEL STUDY OF CHANGES IN WIND WITH ALTITUDE IN A PLANETARY BOUNDARY LAYER

By

N. Godev, D. Iordanov



DDC  
RECEIVED  
JAN 7 1970  
A

~~Distribution of this document is limited. It may be released to the clearinghouse, Department of Commerce, for use to the general public.~~

Reproduced by the  
CLEARINGHOUSE  
for Federal Scientific & Technical  
Information Springfield Va. 22151

## EDITED TRANSLATION

A MODEL STUDY OF CHANGES IN WIND WITH ALTITUDE IN A  
PLANETARY BOUNDARY LAYER

By: N. Godev, D. Iordanov

Source: Bulgarska Akademiya na Naukite. Doklady  
(Bulgarian Academy of Sciences. Reports)  
1967, Vol. 20, No. 9, pp. 911-913

Translated Under: F33657-68-D-0865-P002

English Pages: 3

STATEMENT #2 UNCLASSIFIED

This document is subject to special export controls and each  
transmittal to foreign governments ~~FTD/TTBDR~~ nationals may be  
made only with prior approval of ~~W-F AFB Ohio~~ 45433

THIS TRANSLATION IS A RENDITION OF THE ORIGINAL FOREIGN TEXT WITHOUT ANY ANALYTICAL OR EDITORIAL COMMENT. STATEMENTS OR THEORIES ADVOCATED OR IMPLIED ARE THOSE OF THE SOURCE AND DO NOT NECESSARILY REFLECT THE POSITION OR OPINION OF THE FOREIGN TECHNOLOGY DIVISION.

PREPARED BY:

TRANSLATION DIVISION  
FOREIGN TECHNOLOGY DIVISION  
WP-AFB, OHIO.

# DATA HANDLING PAGE

61-ACCESSION NO. 75-DOCUMENT LOC TP9001149		32-TOPIC TAGS  atmospheric boundary layer, atmospheric turbulence, wind		
62-TITLE A MODEL STUDY OF CHANGES IN WIND WITH ALTITUDE IN A PLANETARY BOUNDARY LAYER				
67-SUBJECT AREA  04, 20				
12-AUTHOR/CO-AUTHORS GODEV, N. ; 16-IORDANOV, D.				18-DATE OF INFO -----67
63-SOURCE BULGARSKA AKADEMIYA NA NAUKITE. DOKLADY (BULGARIAN) FTD-				48-DOCUMENT NO. HT-23-1328-68
				69-PROJECT NO. 72301-78
65-SECURITY AND DOWNGRADING INFORMATION  UNCL, O			64-CONTROL MARKINGS  NONE	97-HEADER CLASH  UNCL
76-REEL FRAME NO. 1889 1651	77-SUPERSEDES	78-CHANGES	49-GEOGRAPHICAL AREA BU	NO OF PAGES 3
CONTRACT NO. F33657-68-D-0865-P002	X REF ACC. NO. 65-AP7034744	PUBLISHING DATE 94-	TYPE PRODUCT TRANSLATION	REVISION FREQ NONE
STEP NO. 02-BU/011/67/020/009/0911/0913			ACCESSION NO.	

## ABSTRACT

(U) <sup>1/2</sup> This paper presents a solution from which the majority of the known solutions are derived as partial cases. These equations and models are used in studying the time-wise changes in wind with height in the planetary boundary layer. Here  $K(z)$  is the kinematic coefficient of turbulent exchange along the  $z$  axis and  $\text{liter}$  is the Coriolis force. (Orig. art. has: 9 formulas.

## A MODEL STUDY OF CHANGES IN WIND WITH ALTITUDE IN A PLANETARY BOUNDARY LAYER

N. Godev, D. Iordanov

(Presented by Academician L. Krystanov on 25 May 1967)

The study of the time-steady variation in wind with altitude in the planetary boundary layer is associated with the solution of the following system of differential equations:

$$\begin{aligned} \frac{\partial}{\partial z} K(z) \frac{\partial u}{\partial z} + l v &= l v_g \\ \frac{\partial}{\partial z} K(z) \frac{\partial v}{\partial z} - l u &= -l u_g \end{aligned} \quad (1)$$

where  $u, v, u_g, v_g$  are the components of the wind and of the geostrophic wind, respectively, along the  $x$ - and  $y$ -axes,  $K(z)$  is the kinematic coefficient of turbulent exchange along the  $z$ -axis and  $l$  is the Coriolis parameter.

The following are the boundary conditions at which System (1) is solved:

$$\begin{aligned} u = v = 0 \text{ when } z = z_0 \\ u, v \text{ limited as } z \rightarrow \infty, \end{aligned} \quad (2)$$

where  $z_0$  is the roughness factor assumed to be constant.

From (1) we easily obtain:

$$\frac{\partial}{\partial z} K(z) \frac{\partial M}{\partial z} - l M = -l M_g \quad (3)$$

while from (2)

$$M = 0 \text{ when } z = z_0 \text{ and } M \text{ limited as } z \rightarrow \infty, \quad (4)$$

where  $M = u + iv$ ;  $M_g = u_g + iv_g$ .

A number of the works examined in the exhaustive review of Reference [1] yield the solution for Eq. (3) for various models of  $K(z)$ . An attempt is made in the present paper to provide a solution from which a large number of the known solutions will be derived as special cases. With this purpose in mind we will seek out solutions to Eq. (3) for the following model of  $K(z)$ :

$$K(z) = \begin{cases} K_1 z^p & \text{when } z \leq h \\ K_2 z^q \text{ or } K_3 z^r & \text{when } z \geq h \end{cases} \quad (5)$$

for the boundary conditions of (4) and the condition when  $z = h$ :

$$\begin{aligned} M(z)|_{z=h-0} &= M(z)|_{z=h+0} \\ K(z) \frac{dM}{dz} \Big|_{z=h-0} &= K(z) \frac{dM}{dz} \Big|_{z=h+0} \end{aligned} \quad (6)$$

Solution (3) for Conditions (4), (5) and (6) is given by the expression

$$M(z) = M_s \left\{ 1 - z^{\frac{1-p}{2}} \frac{(b_2 a_1 - b_1 a_2) H_{\nu}^{(2)} \left( \frac{2\sqrt{-l}}{2-p} \delta_1 z^{\frac{2-p}{2}} \right) - (a_2 a_1 - a_1 a_2) H_{\nu}^{(1)} \left( \frac{2\sqrt{-l}}{2-p} \delta_1 z^{\frac{2-p}{2}} \right)}{(a_1 a_1 - a_2 a_2) b_1(z_0) - (b_1 a_1 - b_2 a_2) a_1(z_0)} \right\} \quad (7)$$

when  $z_0 \leq z \leq h$

$$M(z) = M_s \left\{ 1 - x^{\frac{1-q}{2}} \frac{(a_1 b_2 - a_2 b_1) H_{\mu}^{(2)} \left( \frac{2\sqrt{-l}}{(2-q)b} \delta_2 x^{\frac{2-q}{2}} \right)}{(a_1 a_1 - a_2 a_2) b(z_0) - (b_1 a_1 - b_2 a_2) a_1(z_0)} \right\} \text{ when } z \geq h \quad (8)$$

where

$$\begin{aligned} a_1 &= h^{\frac{1-p}{2}} H_{\nu}^{(2)} \left[ \frac{2\sqrt{-l}}{2-p} \delta_1 h^{\frac{2-p}{2}} \right]; & b_1 &= h^{\frac{1-p}{2}} H_{\nu}^{(1)} \left[ \frac{2\sqrt{-l}}{2-p} \delta_1 h^{\frac{2-p}{2}} \right]; \\ a_2 &= h^{\frac{1-2p}{2}} H_{\nu-1}^{(2)} \left[ \frac{2\sqrt{-l}}{2-p} \delta_1 h^{\frac{2-p}{2}} \right]; & b_2 &= h^{\frac{-2p}{2}} H_{\nu-1}^{(1)} \left[ \frac{2\sqrt{-l}}{2-p} \delta_1 h^{\frac{2-p}{2}} \right]; \\ a_1(z_0) &= z_0^{\frac{1-p}{2}} H_{\nu}^{(2)} \left[ \frac{2\sqrt{-l}}{2-p} \delta_1 z_0^{\frac{2-p}{2}} \right]; & b_1(z_0) &= z_0^{\frac{1-p}{2}} H_{\nu}^{(1)} \left[ \frac{2\sqrt{-l}}{2-p} \delta_1 z_0^{\frac{2-p}{2}} \right]; \\ a_2 &= x_h^{\frac{1-q}{2}} H_{\mu}^{(2)} \left[ \frac{2\sqrt{-l}}{(2-q)b} \delta_2 x_h^{\frac{2-q}{2}} \right]; & a_1 &= \frac{dx}{dz} \Big|_{z=h} \left\{ \left[ \frac{1-q}{2} - \frac{(2-q)\mu}{2} \right] \right. \\ & & & \left. \cdot \frac{x_h^{\frac{1+q}{2}} H_{\mu}^{(2)} \left[ \frac{2\sqrt{-l}}{(2-q)b} \delta_2 x_h^{\frac{2-q}{2}} \right]}{\frac{2\sqrt{-l}}{2-q}} \right\} + x_h^{\frac{1-q}{2}} H_{\mu-1}^{(2)} \left[ \frac{2\sqrt{-l}}{(2-q)b} \delta_2 x_h^{\frac{2-q}{2}} \right] \Big\} \\ \nu &= \frac{1-p}{2-p}; \quad \delta_1 = \sqrt{\frac{l}{K_1}}; \quad \mu = \frac{1-q}{2-q}; \quad \delta_2 = \sqrt{\frac{l}{K_2}} \end{aligned}$$

in the case  $K(z) = K_0 z^q; a=q; x=z; b=1; x_h=h$

in the case  $K(z) = K_0 z^q; a=2; q=3; x=l^z; x_h=l^h$ .

It is not difficult from Expressions (7) and (8) to derive certain of the known solutions. For example, from (7), for the condition  $h \rightarrow \infty$ , we obtain the solution

$$M(z) = M_s \left\{ 1 - \left( \frac{z}{z_0} \right)^{\frac{1-p}{2}} \frac{H_{\nu}^{(2)} \left[ \frac{2\sqrt{-l}}{2-p} \delta_1 z^{\frac{2-p}{2}} \right]}{H_{\nu}^{(2)} \left[ \frac{2\sqrt{-l}}{2-p} \delta_1 z_0^{\frac{2-p}{2}} \right]} \right\}, \quad (9)$$

which was considered in the work by Köhler [2]. From Eq. (9) when  $p = 1$  we obtain the Blinov-Kibel' [3] solution and when  $p = 0$  we obtain the Ekman [sic] spiral. When  $\frac{1-p}{2-p} = r + \frac{1}{2} (r=0, 1, 2, \dots)$  we obtain

the solution considered by Takev [4]. When  $p = 2$  we obtain the solution considered by Takaya [5]. From Eq. (8) when  $h \rightarrow \infty$ , we can obtain: Expression (9) corresponding to the power model of  $K(z)$  for  $a=q; x=z; b=1$  or a known solution [6, 7] for a single-layer exponential model of  $K(z)$ . From (7) and (8) we can derive known two-layer models. Thus, for example, when  $p=1; a=q=0; b_1=1, x=z, K_2=k_1h$  we obtain the Shvets and Yudin [8] model. When  $p=p; a=q=0; x=z; b=1$  and  $K_2=K_1h^p$  we obtain the Berlyand [9] model. When  $p=0; a=q=0; x=z; b=1$  we obtain the Ariyel [10] model. When  $p=p; K_1=\frac{k_1}{h^p}; K_2=\frac{k_2}{h^q}; b=-\frac{q}{h}; a=2; q=3$  we obtain the model developed by Klyuchnikova, Laykhtman and Tseytin [11].

*Geophysics Institute  
Bulgarian Academy of Sciences*

### REFERENCES

1. S.S. Zilitinkevich, D.L. Laykhtman, A.S. Monin. *Izv. AN SSSR. Ser. fiz. okeana i atmosfery. III*, 1967, 3.
2. Kunyl H. Köhler. *Svenska Vetenskapsakad. Handl. Tradje.* 13, 1933, 1.
3. Ye.N. Blinova, I.A. Kibel'. *Dinamicheskaya meteorologiya [Dynamic Meteorology]. Gidrometeoizdat. Moscow, 1937, Part II.*
4. K. Takev. *Khidrologiya i meteorologiya.* 1964, 3.
5. S. Takaya. *Mem. Imp. Marine Obs. Kobe, Japan.* 4, 1930, 1.
6. F. Möller. *Meteorol. Z.* 48, 1931, 2.
7. O.D. Stely. *Arch. Meteorol., Geophys. und Bioklimatol. Ser. A, No. 9*, 1956.
8. M. Shvets, M.I. Yudin. *Tr. GGO.* 1940, No. 31.
9. M.Ye. Berlyand. *Tr. GUGMS.* 1947, Ser. 1, No. 25.
10. N.Z. Ariyel. *Tr. GTO.* 1957, No. 69.
11. L.A. Klyuchinkova, D.L. Laykhtman, G.Kh. Tseytin. *Ibid.* 1965, No. 167.